SOLUTION OF HEAT AND MASS TRANSFER PROBLEMS BY AN ELECTRICAL ANALOG METHOD

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A description is given of a method of electrical modeling of a system of heat and mass transfer equations [1]. A comparison with actual experiment [2, 5] is made.

Unsteady temperature and moisture content fields are described by a system of nonlinear differential equations in partial derivatives [1], which, allowing for sources (or sinks) of heat and moisture, has the form

$$\frac{\partial}{\partial x}\left[(\lambda_{ef}+ra_d\delta_{ef}\varepsilon\gamma_0)\frac{\partial t}{\partial x}\right]-C\gamma_0\frac{\partial t}{\partial\tau}+\frac{\partial}{\partial x}\left[(ra_{def}\varepsilon\gamma_0)\frac{\partial U}{\partial x}\right]+Q=0, \quad (1)$$

$$\frac{\partial}{\partial x}\left[(a_{def}\gamma_0)\frac{\partial U}{\partial x}\right]-\gamma_0\frac{\partial U}{\partial\tau}+\frac{\partial}{\partial x}\left[(a_d\delta_{ef}\gamma_0)\frac{\partial t}{\partial x}\right]+D=0 \quad (2)$$

with the boundary conditions

$$-\lambda_{ef}\left(\frac{\partial t}{\partial x}\right)_s+q_s-r(1-\varepsilon)j_s=0, \quad (3)$$

$$\gamma_0a_{def}\left(\frac{\partial U}{\partial x}\right)_s-\gamma_0a_d\delta_{ef}\left(\frac{\partial t}{\partial x}\right)_s+j_s=0 \quad (4)$$

and with the assigned initial conditions for the temperature and moisture content fields. The coefficients of Eqs. (1)–(4) and the sources (or sinks) of heat and moisture are, in general, variables depending on t, U, the coordinates, the time, etc.

For constant values of the thermophysical properties, Eqs. (1)–(4) may be written in parametric form. The method of calculating the electrical model and the method of modeling do not differ in this case from those described above.

The system of Eqs. (1)–(4) may be solved on two networks of ohmic resistances (see the figure). Since there are terms depending on U in (1), and terms depending on t in (2) and (4), appropriate corrections to the network parameters are made at each step of the solution after determination of t and U. Thus, at each step we carry out a solution by successive approximations. Tests have shown that one or two approximations are sufficient. The method of solution has been described in detail in [3, 4]. Expressions are given below for the parameters of the t-network (solving the heat equation) and of the U-network (solving the moisture equation) for a more general case than in [3, 4].

From the analogy between the equation of the currents passing through the nodes of the t and U networks (see the figure) and Eqs. (1)–(4), written in finite-difference form, we obtain the following expressions from the t-network parameters:

$$R^T_{\lambda i}=\frac{h}{2(\lambda_{ef}+ra_d\delta_{ef}\varepsilon\gamma_0)_i}R^T_N, \quad (5)$$

$$R^T_\tau=\frac{\Delta\tau}{(C\gamma_0)\cdot 2h}R^T_N, \quad (6)$$

$$R^T_U=\frac{(V^T_M-V^T_{0,n})k^ThR^T_N}{2r[(a_{def}\varepsilon\gamma_0)_{1,n}(U_{1,n}-U_{0,n})+(a_{def}\varepsilon\gamma_0)_{2,n}(U_{2,n}-U_{0,n})]}, \quad (7)$$

$$R^T_Q=\frac{(V^T_M-V^T_{0,n})k^T}{2hQ_n}R^T_N. \quad (8)$$

For the U-networks

$$R^U_{a,\gamma_i}=\frac{h}{2(a_{def}\gamma_0)_i}R^U_N, \quad (9)$$

$$R^U_\tau=\frac{\Delta\tau}{\gamma_0\cdot 2h}R^U_N, \quad (10)$$

$$R^U_T=\frac{(V^U_M-V^U_{0,n})k^UhR^U_N}{2[(a_d\delta_{ef}\gamma_0)_{1,n}(t_{1,n}-t_{0,n})+(a_d\delta_{ef}\gamma_0)_{2,n}(t_{2,n}-t_{0,n})]}, \quad (11)$$

$$R^U_D=\frac{(V^U_M-V^U_{0,n})k^U}{2hD_n}R^U_N. \quad (12)$$

Since the method of solution of finite-difference equations on the networks shown in the figure is an implicit method, only the accuracy of solution depends on the values of the space and time integral while the convergence and stability are assured.

Expressions (1)–(4) were written for simplicity for a one-dimensional problem. The derivation of expressions for the network parameters in the case of two- and three-dimensional problems, and the transcript of the equations to other coordinate systems or to dimensionless form is analogous and is carried out in the same way as in [3].

The derivation of expressions for the parameters for nodes located on a body surface does not differ in any way from the derivation for nodes inside a body. Equations (3) and (4) are written in finite-difference form, and from analogy with the expressions for currents according to Kirchhoff's law, we obtain for the t-network

$$R^T_q=\frac{(V^T_M-V^T_{s,n})k^T\lambda_{ef}}{2q(\lambda_{ef}+ra_d\delta_{ef}\varepsilon\gamma_0)}R^T_N, \quad (13)$$

$$R^T_j=\frac{(V^T_M-V^T_{s,n})k^T\lambda_э}{j_s(1-\varepsilon)r\cdot 2(\lambda_{ef}+ra_d\delta_{ef}\varepsilon\gamma_0)}R^T_N. \quad (14)$$

and for the U-network

$$R_j^U = \frac{(V_M^U - V_{s,n}^U)\,k^U}{2j_s}\,R_N^U, \tag{15}$$

$$R_{T,s}^U = \frac{(V_M^U - V_{s,n}^U)\,k^U h}{2(\gamma_0 a_d \delta_{ef})(t - t_s)}\,R_N^U. \tag{16}$$

In the case when

$$q = \alpha(t_s - t_m),$$
$$j = \sigma(U_s - U_m),$$

expressions (14) and (15) will be transformed, respectively, to

$$R_\alpha^T = \lambda_{ef} R_N^T / 2\alpha(\lambda_{ef} + ra_d \delta_{ef}\varepsilon\gamma_0), \tag{17}$$

$$R_\sigma^U = R_N^U / 2\sigma, \tag{18}$$

and potentials will be supplied at the end of the surface resistances calculated from (17) and (18) to the t- and u-networks, corresponding to t_m and U_m (U_m is a fictitious value determined from the equilibrium moisture content of the surrounding medium).

Thus, it is possible, by the method described, to find a solution of the system (1)–(4) by comparatively simple means.

There are difficulties in determining a number of the quantities appearing in (1)–(4) (the heat and mass transfer coefficients, and the power of the external and the internal heat and moisture sources).

From the experimental data available in the literature analytical relations have been obtained to determine the thermophysical properties and certain biochemical characteristics of a granular mass of wheat. *

*This number of the journal carries an article by Zagoruiko on this topic.

The problems of determining the heat and moisture fields of the grain have been solved by the above method.

The results of electrical modeling have been compared with the data from natural experiments [2, 5].

The greatest and average errors of the data from the model did not exceed 12% (and 3%) in temperature, and 6.5% (and 3%) in moisture content, in comparison with the natural exponents [2, 5].

A correction to the heat and mass transfer by filtered air decreases the errors in the investigation by a factor of 2–2.5, in comparison with modeling without allowance for filtration.

The coefficient σ, appearing in (18), was determined from the expression

$$\sigma = \sigma_\rho \frac{\rho_s - \rho_m}{U_s - U_m} = \frac{\alpha}{c_p}\,\frac{\rho_s - \rho_m}{U_s - U_m}, \tag{19}$$

which was obtained from the conditions

$$j = \sigma_\rho(\rho_s - \rho_m),$$
$$j = \sigma(U_s - U_m)$$

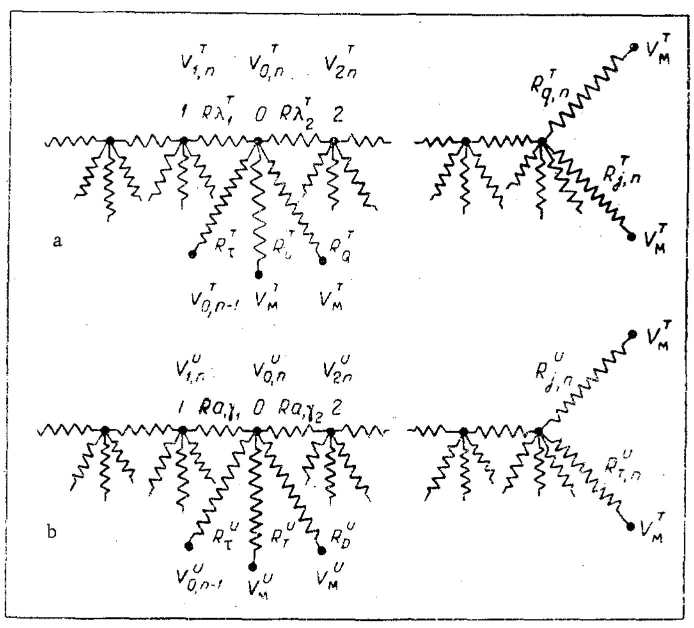
and the Lewis relation

$$\sigma_\rho = \alpha / c_p.$$

The method of electrical modeling described above has the advantage over that described in [6] that at each step of the solution it is possible to introduce corrections which take account of variation of the quantities and of non-linearity.

In principle the "method of composite modeling" [6], in solving problems on model-networks of resistance and capacitances, under the current technique of solution of problems on such networks, is applicable only for problems with constant coefficients.

We have given a more general solution for a heat and mass transfer problem than in [3, 4]; although



Network of resistances for solution of a 1-dimensional heat and mass transfer problem; a is the t-network; b is the U-network.

the method of solution described in [3, 4] and above is more laborious than that in [6], it permits considerable enlargement of the range of investigations of important problems which are practically always nonlinear.

NOTATION

t is the temperature, °C; U is the moisture content, kg moisture/kg dry mass; τ is the time, sec; C is the reduced specific weight heat capacity, j/kg of dry mass · degree; γ_0 is the density of dry mass, kg of dry mass/m^3; c_p is the specific isobaric weight heat capacity of air, j/kg·degree; λ_{ef} is the effective thermal conductivity,* W/m·degree; r is the specific heat of phase transition, j/kg of moisture; a_{def} is the effective diffusion coefficient,* m^2/ /sec; δ_{ef} is the effective thermal-moisture conductivity,* kg of moisture/degree·kg of dry mass; ε is the phase transformation coefficient; Q is the specific power of the heat sources (sinks), W/m^3; D is the specific power of moisture sources (sinks), kg of moisture /m^3·sec; q is the specific surface heat flux, W/m^2; j is the specific surface flux of moisture, kg of moisture/m^2·sec; α is the heat transfer coefficient, W/m^2· degree; σ is the mass transfer coefficient, kg of moisture m^2· sec; ρ is the concentration of vapor in the vapor-air mixture, kg of moisture/kg of moist air; R^t and R^U, respectively, are the electrical resistances of the t- and U-networks; h and $\Delta\tau$, respectively, are the space and time interval ($\tau = n_1\Delta\tau_1 + n_2\Delta\tau_2 + \ldots$), i.e., the time intervals during the solution may be varied; k^T and k^U, respectively, are the scale factors in transition from temperatures and moisture contents to voltages; R_N^T and R_N^U are the standard resistances and scale factors of the transition from natural to electrical quantities; V_M is the maximum or minimum voltage in the problem (when the corresponding term reflects a source of heat or moisture, V_M is the maximum voltage, a sink is the minimum). The subscripts s and m refer a quantity to the surface or to the medium, respectively.

*See the article by Zagoruiko in this same number of the journal.

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